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Major-axis elastic buckling of axially loaded castellated steel columns

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ABSTRACT

Perforated-web steel sections have been used as structural members since the Second World War in an attempt to enhance the flexural behavior without increasing the cost of the material. Nowadays, such sections are widely used in a variety of geometries suitable for various loading conditions. In the current study, the finite element method is used to investigate the major-axis buckling characteristics and associated buckling capacity of axially loaded I-shaped steel columns. Extensive numerical analyses are conducted to evaluate the reduction in buckling capacity of castellated columns due to shear and flexural deformations. Obtained results are used to identify a dimensionless buckling modification factor, η, and the associated equivalent section properties that can be implemented to assess the critical buckling load of the considered columns. The study considers a wide range of practical geometric dimensions, as well as, various columns' end conditions. A simplified procedure is suggested to evaluate the buckling capacity of castellated columns. Charts are developed to enable practitioners to readily estimate the buckling load of such a type of castellated columns more accurately.

1. Introduction

I-shaped steel members represent the basic structural element in majority of structural steel building. Practical considerations may require some of these members to have web openings to allow for the passage and installation of piping, ductworks and/or electrical conduits. Typical web openings that are commonly used in exposed steel structures include hexagonal, octagonal and cellular perforations. The current paper is devoted to studying the influence of hexagonal web castellation on the overall buckling characteristics of axially loaded steel columns as presented in Fig. 1.

Luckily, hexagonal perforations are naturally introduced during the manufacturing of castellated steel members, where the member is cut in a zigzag pattern through its web. The resulting two pieces are then reassembled together by welding as shown in Figs. 2a and b, respectively. Although, the main intent of the castellation process is to produce stiffer I-sections by increasing the web height and providing higher major-axis moment capacity than plain-webbed members of the same weight, it also provides access to services and optimizes the use of the costly structural steel material. These advantages, combined with the significant development in computerized manufacturing equipments, have led to the wide spread use of castellated steel members in various structural applications.

The first attempt to quantify the reduction in the compressive strength of axially loaded plain-webbed columns due to buckling was carried out by Euler in 1774 [1]. He theoretically determined the elastic buckling load P_e of a pin-ended column having a length L and second moment of area I and is made of a linearly elastic material with Young's modulus E as

$$P_e = \frac{\pi^2 EI}{L^2} \tag{1}$$

The above equation considers only the flexural stiffness and deformations of the column while the shear deformations are ignored. Euler formula was then modified by Engesser [2,3] to include the effect of shear deformations on the compressive capacity of prismatic columns in accordance with the following Engesser's formula:

$$P_{cr} = \frac{P_e}{1 + (nP_e/AG)} \tag{2}$$

where A is the area of the column's cross-section, ν is Poisson's ratio, G is the shear modulus of the column material ($G = E/[2(1+\nu)]$) and n is a numerical factor depending on the shape of the column's cross-section. The accuracy of Engesser's formula in considering the shear effect on the elastic stability of plain-webbed columns was verified by Nanni [4] and Ziegler [5]. The influence of the shear deformations on reducing the buckling capacity, especially for short columns, was highlighted by Ziegler [5]. More investigations on the effect of shear deformations were conducted by Gjelsvik [6] using Engesser's formula. The study concluded the appropriateness of Engesser's formula for columns

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Fig. 1. A photo of castellated columns in structural skeletons (by permission of Westok Limited, UK).

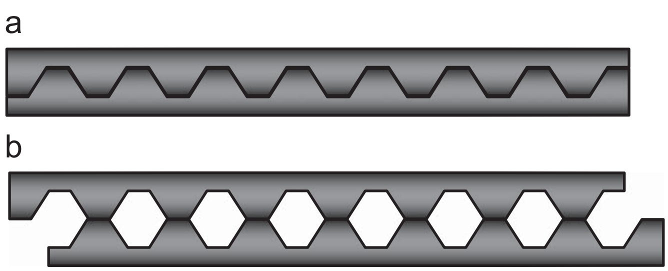

Fig. 2. Manufacturing of steel members with hexagonal web-castellation. (a) Typical cut of castellated member. (b) Reassembled castellated member.

that can be modeled as continuous Timoshenko shear beams [7] where plane sections are assumed to remain plane after deformation, but do not remain normal to the displaced axis of the column [8].

On the contrary to plain-webbed columns, shear deformations have a more pronounced impact on the buckling capacity of built-up columns [1]. The vital rule that shear plays in reducing the buckling capacity of built-up columns was evident following the catastrophic failure of the first Quebec Bridge in 1907 [9,10]. An extensive literature survey has been conducted by Elmahdy [11] on the effect of shear on the buckling capacity of built-up columns. Reported investigations revealed that built-up columns exhibit reduced shear stiffness resulting in an increase in lateral deflection and consequently a reduction in the compressive load carrying capacity. Timoshenko and Gere [1] derived approximate formulae that account for the shear flexibility of built-up columns with battened, laced or perforated cover plates as interconnectors. To the best of the authors' knowledge, no similar formula is available to provide a reliable estimate of the critical buckling load of castellated columns.

The previous review demonstrates the lack of information related to the influence of shear deformations on the buckling capacity of castellated columns. This may be attributed to the limited production of such structural members in the past. The current paper presents a comprehensive study to quantify the effect of shear deformations on the compressive capacity of castellated column when buckles about the cross-section major axis. The equivalent slenderness ratio for the practical range of geometric dimensions and boundary conditions of castellated columns is assessed to help characterizing the compressive response of such columns.

2. Design philosophy

Current international design codes adopt the equivalent slenderness ratio approach to evaluate the critical buckling load of built-up columns. The thirteenth edition of the 2005 AISC Steel Construction Manual [12] provides the following formulae to evaluate the modified column slenderness ratio $(kL/r)_m$ that corresponds to the equivalent cross-section of built-up column buckled about an axis perpendicular to the plane of the battens:

(a) For intermediate connectors that are snug-tight bolted

$$\left(\frac{kL}{r}\right)_m = \sqrt{\left(\frac{kL}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2} \tag{3.a}$$

(b) For intermediate connectors that are welded or pretensioned bolted

$$\left(\frac{kL}{r}\right)_m = \sqrt{\left(\frac{kL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{1+\alpha^2}\left(\frac{a}{r_{ib}}\right)^2} \tag{3.b}$$

where $(kL/r)_o$ is the column slenderness of the entire built-up member acting as a unit in the buckling direction being considered, a is the distance between battens, r_i is the minimum radius of gyration of individual component, r_{ib} is the radius of gyration of individual component relative to its centroidal axis parallel to the member axis of buckling, α is the separation ratio $\alpha = h/2r_{ib}$ and h is the distance between centroids of individual components perpendicular to the member axis of buckling. To avoid the interaction between local buckling of each component over its free length between battens and the global buckling of the column, the ratio (a/r_i) of each component should not exceed three-fourth times the governing slenderness ratio of the built-up member.

Meanwhile, the code of practice for design rolled and welded sections of the British Standard BS5950-1:2000 [13] requires the equivalent slenderness ratio of welded or bolted battened columns λ_b about the axis perpendicular to the plane of the battens to be calculated in accordance with the following equation:

$$\lambda_b = \sqrt{\lambda_m^2 + \lambda_c^2} \geqslant 1.4\lambda_c \tag{4}$$

where λ_c is the slenderness ratio of the component between end welds or end bolts of adjacent battens based on its minimum radius of gyration and λ_m represents the slenderness ratio of the entire member $= L_E/r$, in which, L_E is the effective buckling length of the entire column and r is radius of gyration of the entire member parallel to the axis of buckling. The British Standard BS5950-1:2000 [13] enforces the condition that the slenderness λ_c should not exceed 50 to eliminate the possibility of having interaction between local buckling of each component over its free length between battens and the global buckling of the column.

Nonetheless, no direct procedure is provided in current design codes for evaluating the compressive load-carrying capacity of columns with web perforations. The current study aims at developing a reliable but simple procedure to identify the critical buckling load for a wide range of practical dimensions of castellated columns with various boundary conditions. This is achieved in a similar manner to the current procedure recommended by international design codes through the implementation of the equivalent slenderness ratio approach.

3. Development of the finite element model

Numerical modeling and analysis of castellated columns is conducted using ANSYS [14], a general purpose finite element software package, to determine the critical elastic buckling load and the associated mode of failure of such castellated columns. Three-dimensional (3D) 6-noded and 8-noded structural solid elements (SOLID45), with three translational degrees of freedom at each node, are used to model the geometrical details of analyzed columns. Typical geometry of modeled castellated columns is presented in Fig. 3. According to this illustration, the typical spacing between castellation is 1.5d, center-to-center, where d represents the diameter of the circle enclosing the hexagonal perforation. The gain in the depth of the expanded section, relative to the original depth, is estimated as 0.433d.

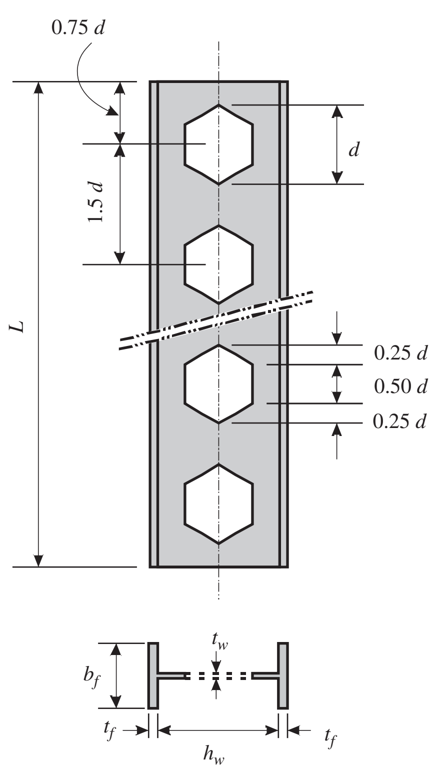

Fig. 3. Geometry of a typical castellated column.

3.1. Modeling assumptions

The main assumptions employed in the conducted investigation are summarized below:

1. all columns are assumed to have a linear elastic material with a modulus of Elasticity $E = 2 \times 10^5$ MPa and Poisson's ratio $\nu = 0.3$;
2. the considered I-shaped column is defined by its length L, flange width b_f, flange thickness t_f, web width h_w and web thickness t_w, as shown in Fig. 3;
3. hexagonal web perforations are uniformly spaced at distance s along the column axial direction (Fig. 3), the L/s ratio is controlled to allow for the generation of integer number of perforations along the column length;
4. the dominant failure mode is in-plane buckling of the column accompanied by major-axis bending;
5. various end conditions are implemented to simulate different boundary conditions;
6. the column is axially loaded with concentrated loads applied at its ends and
7. the column buckling load is calculated following the small deformation theory and is, therefore, obtained by solving an Eigen-value problem.

3.2. Modeling assumptions

Flanges are modeled with two elements across the thickness (t_f) and 10 elements along the width (b_f). Meanwhile, web plates are modeled with three elements across the thickness (t_w). The number of elements used to model other parts of the column cross-section are summarized in Table 1. For the range of geometrical dimensions assumed in the current study, the limits presented in Table 1 and Fig. 4a for the number of elements are found to provide convergence of the buckling load based on several trial buckling analyses that have been conducted using higher and less number of elements.

As a result of the symmetry in the column geometry, loading and response, only half of the column is modeled as shown in Fig. 4b. The mesh arrangement used for the finite element analysis is shown in Fig. 4c. The buckling analysis accounts for the effect of the boundary conditions on the column capacity by considering four end conditions; pinned–pinned (P–P), fixed–pinned (F–P), fixed–fixed (F–F) and fixed–free (F–Free). It should be mentioned that to avoid any secondary stresses resulting from local deformations at the column ends (especially close to the hole), all the nodes at each of the column ends are rigidly constrained to ensure that plane sections remain plane after deformation. The appropriate end conditions, either for a pin or a fixation, are then applied by imposing the appropriate restraints at the node located at the centroid of the cross-section at the column end. To achieve these idealized end conditions, dummy flexible shell elements with rotational and translational nodal degrees of freedom are added to the cross-sections at the column ends.

3.3. Validation of the developed model

The performance of the developed 3D finite element model is validated by evaluating the critical buckling load of I-shaped plain-webbed columns and comparing it to the compressive capacity of Engesser's analytical formula, Eq. (2), that accounts for shear deformations in plain-webbed columns [2,3]. It should be noted that the validation stage accounts also for various boundary conditions other than the pinned–pinned case studied by Engesser. The influence of the column boundary conditions on

Table 1
Controlling parameters of finite element meshing.

Mesh parameter	Description	Minimum number of elements	Maximum number of elements
n_1	Number of elements along the spacing between perforations (s)	8	20
n_2	Number of elements along the web clear height (h_w)	12	20
n_3	Number of elements along each side of the hexagonal web perforation	4	–
n_4	Number of elements in half the solid web post between perforations $(s-d)/2$	4	10
n_5	Number of elements in the stem of the T-section located above and below each web perforation $(h/2-d/2-t_f)$	2	6

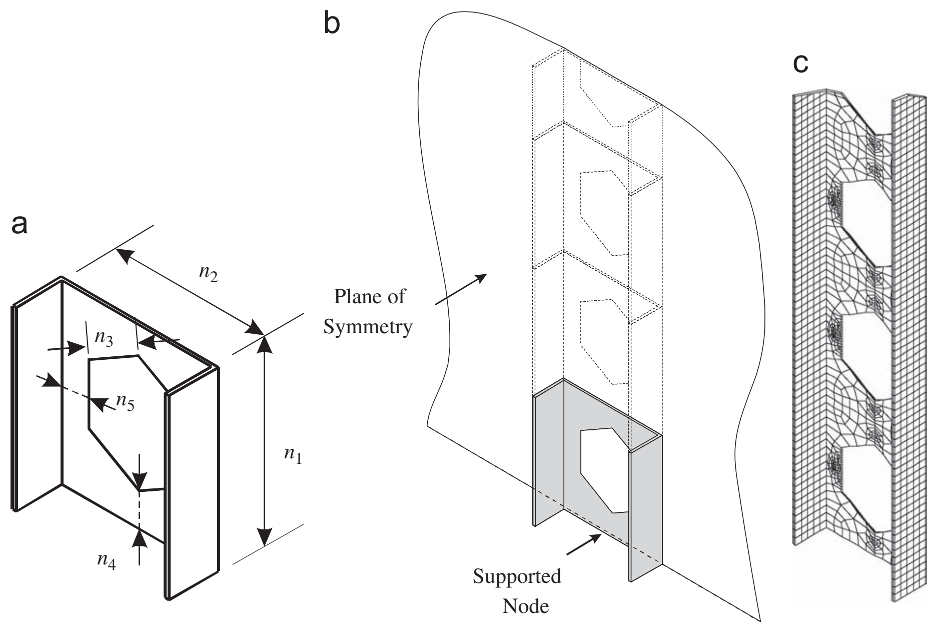

Fig. 4. Sample finite element mesh and controlling parameters for analyzed castellated columns.

the buckling load is accounted for by considering various end conditions including (P–P), (F–P), (F–F) and (F–Free) columns for which $k = 1.0$, 0.7, 0.5 and 2.0, respectively. The effective buckling length factor (k) is incorporated in the load calculation which yields the following expression of the analytical buckling load P_{an}

$$P_{an} = \frac{P_e}{1+(nP_e/AG)} = \frac{EI}{(kL/\pi)^2 + (nEI/AG)} \tag{5}$$

in which $n \approx A/A_w$, where A is the area of the cross-section and A_w is the area of the web $A_w = h_w t_w$. The validation process considers about 100 different columns that are modeled using the developed finite element model to assess the buckling load P_{FE} that is compared to its analytical counterpart P_{an}. A sample of obtained results is presented in Table 2 for columns having $h_w/t_w = 45$, $kL/h_w = 30$ and $b_f/t_f = 4$ and 25, respectively. Quantitative comparison between the two sets of results shows an absolute maximum relative error of about 1.65% for the case of a fixed–fixed column with $I_f/I_w = 13.1$ and $b_f/t_f = 4$, where I_f and I_w are the second moment of area of the flange and web relative to the column centroidal axis perpendicular to the plane of buckling, respectively, and are defined as

$$I_f = \frac{b_f t_f^3}{12} + (b_f t_f)\left(\frac{h_w + t_f}{2}\right)^2 \tag{6.a}$$

$$I_w = \frac{t_w h_w^3}{12} \tag{6.b}$$

Meanwhile, the absolute minimum relative error is less than 0.02% which corresponds to the case of a fixed–free column

Table 2
Validation of the finite element model ($kL/h_w = 30$).

End conditions	b_f/t_f	I_f/I_w	P_{an} (kN)	P_{FE} (kN)	Relative error $\lvert P_{an}-P_{FE}/P_{an}\rvert$%
P–P	4	0.6	1866.6	1854.4	0.65
		1.7	3836.9	3816.2	0.54
		2.4	5239.6	5215.4	0.46
		4.3	8926.2	8905.3	0.23
		6.7	13830.6	13846.6	0.12
		9.6	19975.8	20104.8	0.65
		13.1	27350.3	27731.7	1.39
F–P	4	0.6	1866.6	1862.6	0.21
		1.7	3836.9	3829.7	0.19
		2.4	5239.6	5231.2	0.16
		4.3	8926.2	8922.5	0.04
		6.7	13830.6	13856.6	0.19
		9.6	19975.8	20090.6	0.57
		13.1	27350.3	27666.9	1.16
F–F	4	0.6	1901.2	1864.9	0.09
		1.7	3797.6	3834.6	0.06
		2.4	5113.5	5238.8	0.02
		4.3	8483.0	8940.2	0.16
		6.7	12826.3	13895.7	0.47
		9.6	18114.8	20167.2	0.96
		13.1	24302.1	27801.9	1.65
F–Free	4	0.6	1901.2	1854.3	0.66
		1.7	3797.6	3816.8	0.52
		2.4	5113.5	5216.9	0.43
		4.3	8483.0	8909.8	0.18
		6.7	12826.3	13856.6	0.19
		9.6	18114.8	20122.7	0.74
		13.1	24302.1	27759.8	1.50

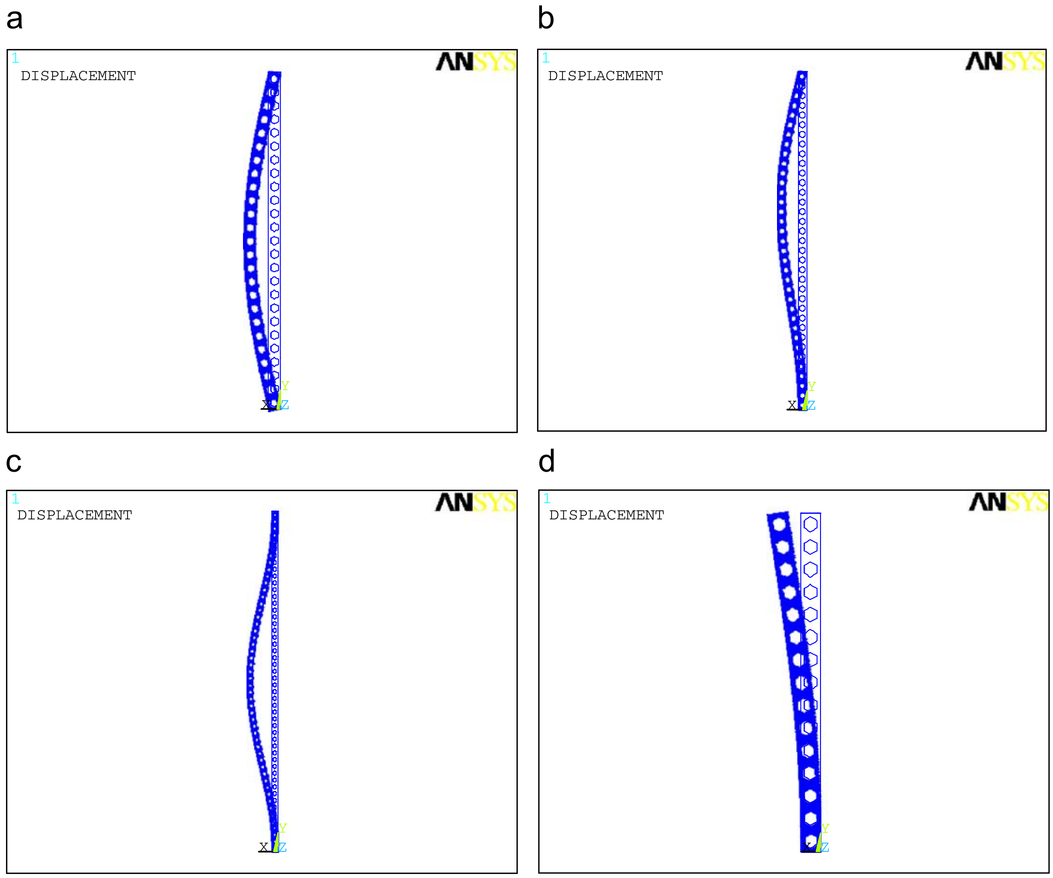

Fig. 5. Buckling modes of castellated columns for various boundary conditions. (a) Pinned–pinned (P–P), (b) fixed–pinned (F–P), (c) fixed–fixed (F–F) and (d) fixed–free (F–Free).

having $I_f/I_w = 5.4$ and $b_f/t_f = 4$. The higher error values associated with bigger I_f/I_w ratios are attributed to the higher order of the error associated with the approximate (n) parameter in Engesser's formula. The close agreement shown between the analytical solution and numerical results validates the accuracy of the developed finite element model in predicting the critical buckling load of axially loaded columns, including the effect of shear deformations.

4. Numerical analysis of castellated columns

The validated 3D finite element model is utilized to perform an extensive parametric analysis to identify the buckling characteristics of castellated columns. A wide range of geometrical properties of analyzed sections is selected to cover, and extend beyond, the practical range of dimensions of such columns. An extensive survey is conducted to identify the commonly used dimensions in the European and North American markets. In addition, the survey considers the recommendations of the European Committee for Standardization [15,16]. Based on the survey outcomes, sections with the relative flange-to-web flexural stiffness (I_f/I_w) that ranges between around 0.60 and 17.0 and b_f/t_f-ratio that varies from 4 to 25 are considered. The size of web castellation relative to the web height (d/h_w) is considered to vary in the range 0.5–0.9.

It is crucial to carefully identify the appropriate length of columns considered in the current study. This is attributed to the fact that short columns may have a reduced buckling capacity, relative to longer columns with the same cross-section dimensions, as a result of the higher shear deformations induced in short columns [5]. In addition, significant shortening of analyzed columns may trigger unfavorable failure mode resulting from the local buckling of the cross-section elements combined with the overall buckling of the entire column. The coupled local–global instabilities would result in a highly unstable post-buckling behavior and, therefore, the buckling capacity of a column experiencing interaction between local and global buckling is significantly reduced [17]. The focus of the current study is to explore the global buckling characteristics of castellated columns and therefore, it is essential to eliminate any possibility of interaction between local and global buckling modes. In the employed finite element model, the symmetry boundary conditions implemented require restraining the out-of-plane web deformations and, therefore, local web buckling is prevented. As this special case does not apply to the flange plates, flange local buckling may be avoided by ensuring that overall column buckling precedes local failure in its flange plates in accordance with the following criterion:

$$f_{cr-overall} < f_{cr-fl} \tag{7}$$

in which, $f_{cr-overall}$ is the critical compressive stress associated with elastic buckling of the entire column. Meanwhile, f_{cr-fl} represents the elastic buckling stress of the flange plates given by [1]

$$f_{cr-fl} = \psi \frac{\pi^2 E}{12(1-\nu^2)(b_f/2t_f)^2} \tag{8}$$

where ψ is a constant that depends on the boundary conditions and the length-to-width ratio of the flange plate. A value of $\psi = 0.7$ is considered to resemble the rigid-free boundary conditions of flange plate [13]. The elastic buckling stress of the entire column is based on the Euler's basic equation

$$f_{cr-overall} = \frac{\pi^2 E}{(kL/r)^2} \tag{9}$$

where (kL/r) represents the overall slenderness ratio of the column in which, kL is the effective buckling length of the entire column and r is the radius of gyration parallel to the axis of buckling. Eqs. (7)–(9) are used to identify the overall slenderness ratio (kL/r) at which the overall column buckling precedes the flange local buckling. For over 200 plain-webbed columns considered with various cross-sectional dimensions and end conditions, analyses reveal that a critical value of $(kL/r \geqslant 50)$ satisfies the condition presented in Eq. (7). This threshold value is even magnified by a factor of 1.67 (which corresponds to $kL/h_w = 30$) to ensure that overall buckling is the only dominant buckling mode of analyzed columns. All reported finite element analyses and their results are for $kL/h_w = 30$ unless otherwise noted. A detailed general approach is presented in the next section to explain extending the results associated with the particular value of $kL/h_w = 30$ to correspond to any other (kL/h_w) ratio.

5. Identification of buckling characteristics

As it was alluded to before, the basic objective of this paper is to enable designers to assess the buckling capacity of castellated columns in a similar manner to that adopted by current international design codes. This requires determination of the critical buckling load in terms of the equivalent slenderness ratio which is a unique characteristic of each particular castellated column depending on its geometrical parameters. The critical buckling load can be evaluated in accordance with the following equation:

$$P_{cr} = \frac{\pi^2 EA}{(kL/r)_{eq}^2} \tag{10}$$

where A is the gross area of the column cross-section without considering the reduction in area due to web castellation (i.e., $A = 2b_f t_f + h_w t_w$), and $(kL/r)_{eq}$ stands for the equivalent slenderness ratio of the castellated column that is to be used in Eq. (10) to result in an accurate estimate of the critical buckling load P_{cr}. In other words, the equivalent slenderness ratio is a design parameter that reflects the influence of web castellation on the buckling capacity through consideration of the change in flexural and shear deformations of castellated columns from plain-webbed columns. The equivalent slenderness ratio can be expressed as a function of the traditional slenderness ratio through the dimensionless buckling modification factor, η, as

$$\eta = \frac{(kL/r)_{eq}}{(kL/r)} \tag{11}$$

The procedure followed to determine the buckling modification factor η involves the following steps:

1. Identification of the critical load P_{cr} using the developed finite element model.
2. Eq. (10) is applied to calculate the corresponding equivalent slenderness ratio $(kL/r)_{eq}$.
3. Finally, the buckling modification factor η is obtained based on Eq. (11).

Samples of the buckling modes resulting from the finite element analysis are provided in Figs. 5a–d for castellated columns with (P–P), (F–P), (F–F) and (F–Free) boundary conditions, respectively.

A set of finite element analyses is conducted on various castellated columns to investigate the influence of web dimensions (h_w, t_w), flange dimensions (b_f, t_f), size of web castellation relative to the web height (d/h_w) and theoretical buckling length, for various end conditions, relative to web height (kL/h_w) on the critical buckling load of castellated columns. Preliminary results indicate that the finite element analyses of castellated columns with similar controlling parameters kL/h_w, I_f/I_w, d/h_w, h_w/t_w and b_f/t_f produce the same modification factor η. This observation is shown in the sample results summarized in Table 3 for various columns with $(kL/h_w = 30)$. In view of these results, it is clear that equal modification factor η can be applied for castellated columns with equal I_f/I_w, d/h_w, h_w/t_w and b_f/t_f values irrespective of the specific dimensions of the web plate (h_w, t_w) and the flange plate (b_f, t_f) of such columns. As a result, a significant reduction in the amount of numerical results to be presented can be achieved since no correlation exists between the modification factor and the specific dimensions of cross-section elements. Variation of the modification factor η with various controlling parameters that characterize the geometry of the column and the configuration of

Table 3
Variation of the buckling modification factor η for castellated columns with different cross-section dimensions and $kL/h_w = 30$.

I_f/I_w	d/h_w	Web dimensions (mm)		Flange dimensions (mm)		h_w/t_w	b_f/t_f	η
		h_w	t_w	b_f	t_f			
0.6	0.5	100	5	20	5	20	4	1.023
		400	20	80	20			1.023
		200	10	100	4		25	1.024
		400	20	200	8			1.024
15.0	0.5	200	10	200	50	20	4	1.081
		800	40	800	200			1.081
		100	5	250	10		25	1.093
		200	10	500	20			1.093
0.6	0.6	100	5	20	5	20	4	1.034
		400	20	80	20			1.034
		200	10	100	4		25	1.035
		400	20	200	8			1.035
15.0	0.6	200	10	200	50	20	4	1.088
		800	40	800	200			1.088
		100	5	250	10		25	1.102
		200	10	500	20			1.102
0.6	0.7	100	5	20	5	20	4	1.050
		400	20	80	20			1.050
		200	10	100	4		25	1.053
		400	20	200	8			1.053
15.0	0.7	200	10	200	50	20	4	1.094
		800	40	800	200			1.094
		100	5	250	10		25	1.115
		200	10	500	20			1.115
0.6	0.8	100	5	20	5	20	4	1.075
		400	20	80	20			1.075
		200	10	100	4		25	1.079
		400	20	200	8			1.079
15.0	0.8	200	10	200	50	20	4	1.099
		800	40	800	200			1.099
		100	5	250	10		25	1.134
		200	10	500	20			1.134
0.6	0.9	100	5	20	5	20	4	1.108
		400	20	80	20			1.108
		200	10	100	4		25	1.118
		400	20	200	8			1.118
15.0	0.9	200	10	200	50	20	4	1.107
		800	40	800	200			1.107
		100	5	250	10		25	1.160
		200	10	500	20			1.160

web castellation such as I_f/I_w, d/h_w, h_w/t_w, b_f/t_f and kL/h_w along with the associated degree of dependency is discussed in details in the following subsections.

5.1. Variation of the modification factor η with web dimensions (h_w/t_w)

Results of the parametric analysis reveal that for castellated columns with equal (I_f/I_w), (d/h_w) and (b_f/t_f) ratios, the aspect ratio of the web plate (h_w/t_w) has insignificant effect on the buckling modification factor η. A sample of the results is presented in Table 4 for 16 different columns that represent extreme geometrical parameters where d/h_w-ratio equals 0.5 and 0.9 and b_f/t_f-ratio is 4 and 25. Half of the considered columns are assigned I_f/I_w-ratio of 0.6, while the other half corresponds to a higher I_f/I_w-ratio of 15.0. Analyzed cases are sorted in Table 4 in such a way that for each two consecutive rows, the mentioned controlling parameters I_f/I_w, d/h_w and b_f/t_f are equal, while the aspect ratio of the web plate (h_w/t_w) is unequal with assumed extreme values of 5 and 45, respectively. Tabulated results indicate that the value for the buckling modification factor η is almost identical for the two extreme values of (h_w/t_w) with a maximum difference in the order of 2.5%. This trend is thought to be related to the equal reduction in the shear and flexural stiffness of the different cross-sections with equal relative flexural stiffness (I_f/I_w) and castellation effect (d/h_w) with minor effect of (b_f/t_f).

5.2. Variation of the modification factor η with flange dimensions (b_f/t_f)

Finite element results show that the buckling modification factor η is linearly proportional to the increases in the b_f/t_f-ratio. The obtained variation trend is shown in Figs. 6a–c for castellated columns having $d/h_w = 0.5$, 0.7 and 0.9, respectively. As a result of the observed linear trend, the buckling modification factor η that corresponds to any (b_f/t_f) ratio other than 4 and 25 can be easily assessed by linearly interpolating between the two extreme values provided in this study for (b_f/t_f) = 4 and 25 as explained in more details in the next subsection and Figs. 7 and 8.

Table 4
Variation of the buckling modification factor η with web aspect ratio (h_w/t_w) for ($kL/h_w = 30$).

I_f/I_w	d/h_w	Web dimensions (mm)		Flange dimensions (mm)		h_w/t_w	b_f/t_f	η
		h_w	t_w	b_f	t_f			
0.6	0.5	50	10	20	5	5	4	1.023
		450	10	60	15	45		1.023
0.6	0.5	50	10	50	2	5	25	1.023
		450	10	150	6	45		1.024
15.0	0.5	50	10	100	25	5	4	1.084
		450	10	300	75	45		1.082
15.0	0.5	50	10	250	10	5	25	1.117
		450	10	750	30	45		1.088
0.6	0.9	50	10	20	5	5	4	1.098
		450	10	60	15	45		1.111
0.6	0.9	50	10	50	2	5	25	1.113
		450	10	150	6	45		1.119
15.0	0.9	50	10	100	25	5	4	1.109
		450	10	300	75	45		1.112
15.0	0.9	50	10	250	10	5	25	1.162
		450	10	750	30	45		1.166

5.3. Variation of the modification factor η with relative flexural stiffness (I_f/I_w)

Parametric analysis results indicate that relative flexural stiffness (I_f/I_w) is a major factor to consider when evaluating the buckling capacity of castellated columns due to the strong correlation between such a factor and the buckling modification factor η. Figs. 7 and 8 present the variation of the buckling modification factor η with the dimensionless parameters (I_f/I_w) and (d/h_w) for (b_f/t_f) = 4 and 25, respectively. Both graphs correspond to a reference slenderness value of $kL/h_w = 30$ to avoid interaction between overall and local instabilities as previously explained in Section 4. Although graphs are provided only for two particular values of (b_f/t_f) = 4 and 25, the buckling modification factor η that corresponds to any (b_f/t_f) ratio, other than 4 and 25, can be readily assessed by linearly interpolating between the two values of η provided in Figs. 7 and 8.

The general trend shown by the plots reveal also that more reduction in the buckling capacity is associated with the increase in the relative flexural stiffness (I_f/I_w) due to the associated reduction in the shear stiffness of the web and consequently the column. The plots show also that the optimum relative flexural stiffness (I_f/I_w) that is the (I_f/I_w) value that corresponds to the least reduction in the buckling capacity, is centered around a value of 2. This value is, in fact, the typical average (I_f/I_w) value for commonly used steel sections in practice. Both figures demonstrate also that columns with bigger web castellation, i.e.; higher (d/h_w) ratio, experience more reduction in their buckling capacity, associated with higher buckling modification factor η, as a result of the higher reduction in their shear stiffness. For columns with smaller (d/h_w) ratio, the buckling modification factor η approaches unity which is the typical value to be used for plain-webbed long columns, where the shear deformations are minimal relative to those induced in castellated columns.

5.4. Variation of the modification factor η with column length

The buckling modification factors η reported in the previous subsection, Figs. 7 and 8, apply to castellated columns with a reference $kL/h_w = 30$ to ensure their buckling capacity is controlled by their overall instability only. To use these η factors to determine the reduced critical buckling load associated with an arbitrary (kL/h_w) ratio, it is essential to separate the effect of shear deformations on the buckling capacity of castellated columns from other parameters. Engesser's formula (Eq. (2)) can be rewritten as

$$P_{cr} = \frac{P_e}{1 + (nP_e/AG)} \text{ or } \frac{n}{AG} = \left(\frac{1}{P_{cr}} - \frac{1}{P_e}\right) \tag{12}$$

It is important to notice that the term $[nP_e/AG]$ represents the effect of shear deformations on the critical buckling load of the castellated column. The term $[n/AG]$ is solely dependent on the cross-section geometry and dimensions, while P_e is the Euler buckling load that is not a function of the shear effect, but rather is dependent on the column's length. In the current study, a conservative assumption is adopted where columns with (kL/h_w) that exceeds 150 (i.e., $kL/r > 250$) are considered long enough for the influence of shear deformations to vanish. The Euler load P_e can be defined as

$$P_e = \frac{\pi^2 EI_{eq}}{(kL)^2} = \frac{\pi^2 EI}{\eta^2 (kL)^2} \tag{13}$$

where I_{eq} is the equivalent second moment of area of the column's cross-section, which is a key parameter that needs to be

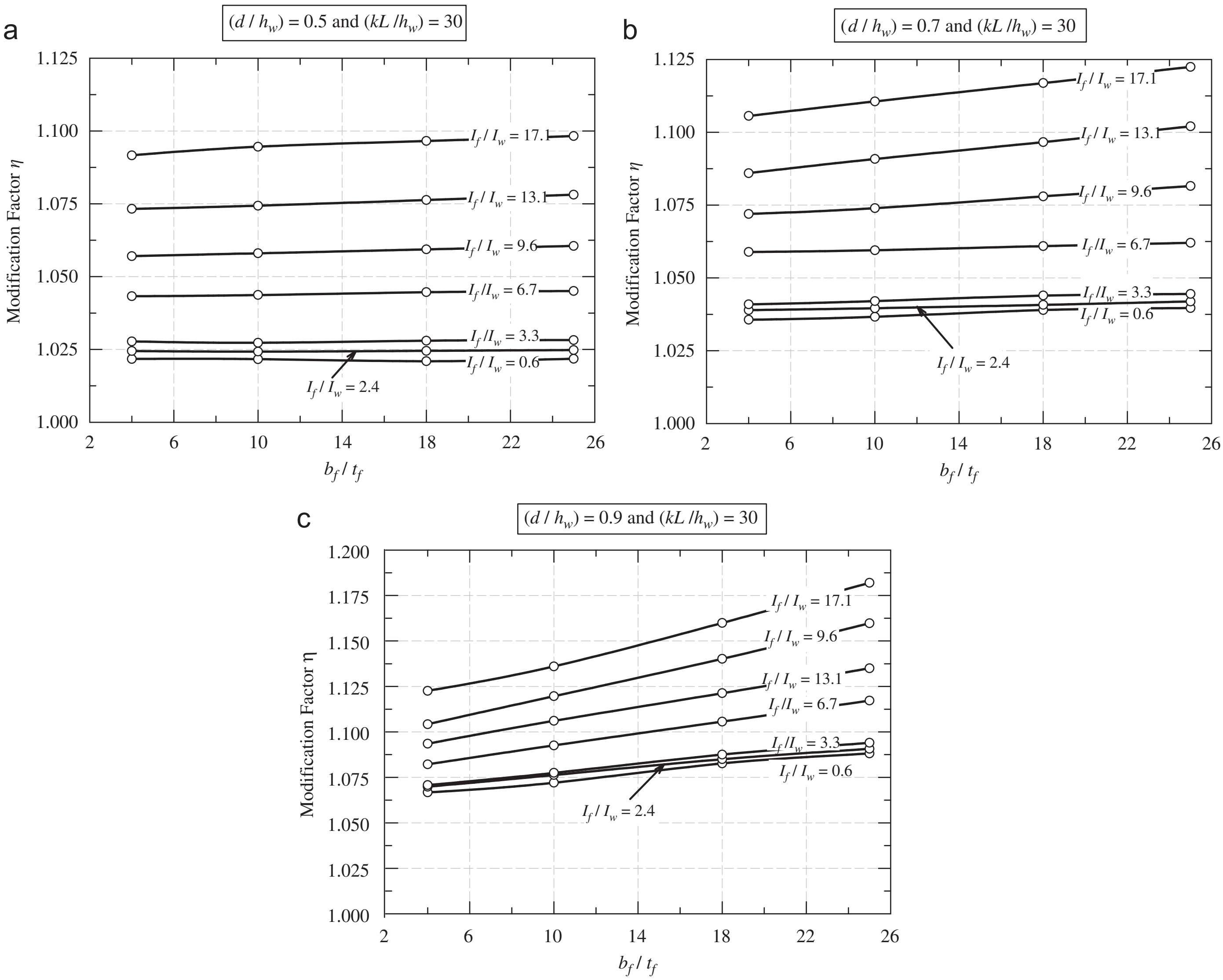

Fig. 6. Variation of the buckling modification factor η with (b_f/t_f) for different (d/h_w) values. (a) Case of $d/h_w = 0.5$; (b) Case of $d/h_w = 0.7$; (c) Case of $d/h_w = 0.9$.

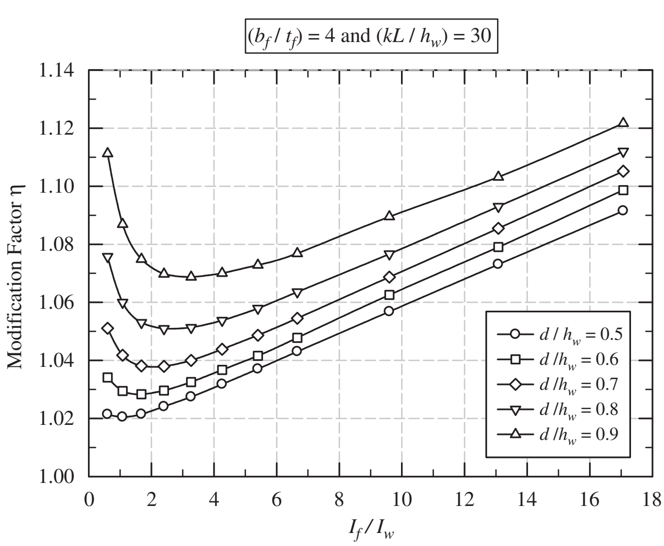

Fig. 7. Variation of the buckling factor η with column dimensions for $(b_f/t_f = 4)$.

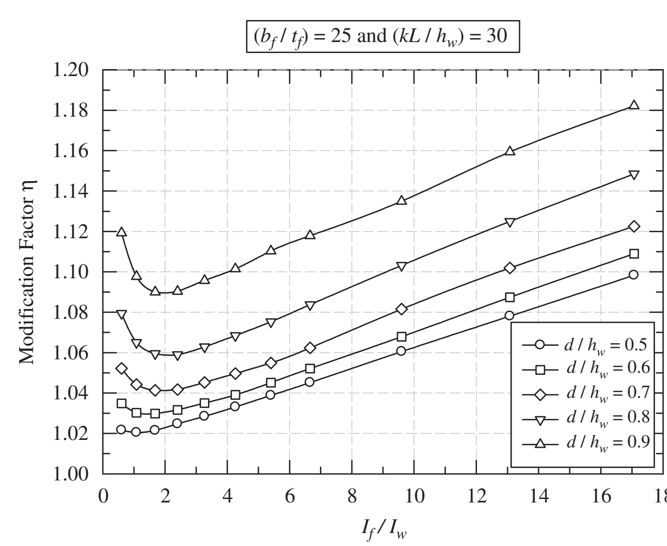

Fig. 8. Variation of the buckling factor η with column dimensions for $(b_f/t_f = 25)$.

evaluated. It is clear that I_{eq} depends primarily on the cross-sectional properties and more specifically on the appropriate net properties of the castellated web. The identification of the reduced section due to castellation is a trial and error process that is carried out to identify the appropriate equivalent cross-section to be used in the calculation of I_{eq}. Several trials with very long columns (i.e., $kL/h_w > 150$) are conducted and results from the finite element are compared with Eq. (13). This process indicates that considering a virtual web with a central part of size (0.806d) removed has appropriately reduced the second moment of area of the cross-section under the following two basic conditions:

- The critical buckling load, P_{cr}, evaluated based on Eq. (13) should be equal to the buckling load, P_{FE}, identified by ANSYS [14]. This implies that the proposed procedure should predict the same value that would result from a detailed 3D finite element analysis of castellated columns.
- The dimensionless buckling modification factor η should approach a value of unity for columns that are significantly long relative to the size of their cross-section [1,5], where the column behaves like a long bar with no shear deformations induced and, therefore, no reduction in the buckling load is expected.

To derive an expression for the dimensionless buckling modification factor η_i for any castellated column length L_i, Eq. (12) can be written for two column lengths; first for the reference length L (corresponding to $kL/h_w = 30$) and secondly for any arbitrary length L_i.

$$\frac{n}{AG} = \left(\frac{1}{P_{cr}} - \frac{1}{P_e}\right) = \left(\frac{1}{P_{cr}} - \frac{1}{P_e}\right)_i = \frac{\eta^2 (kL)^2}{\pi^2 EI} - \frac{(kL)^2}{\pi^2 EI_{eq}} = \frac{\eta_i^2 (kL_i)^2}{\pi^2 EI} - \frac{(kL_i)^2}{\pi^2 EI_{eq}} \tag{14}$$

Eq. (14) can be simplified to give the following general expression for the buckling modification factor η_i for a castellated column with length L_i

$$\eta_i = \sqrt{\left(\frac{I}{I_{eq}}\right) + \left(\frac{L}{L_i}\right)^2 \left(\eta^2 - \frac{I}{I_{eq}}\right)}, \text{ where } I_{eq} = I - \frac{t_w (0.806d)^3}{12} \tag{15}$$

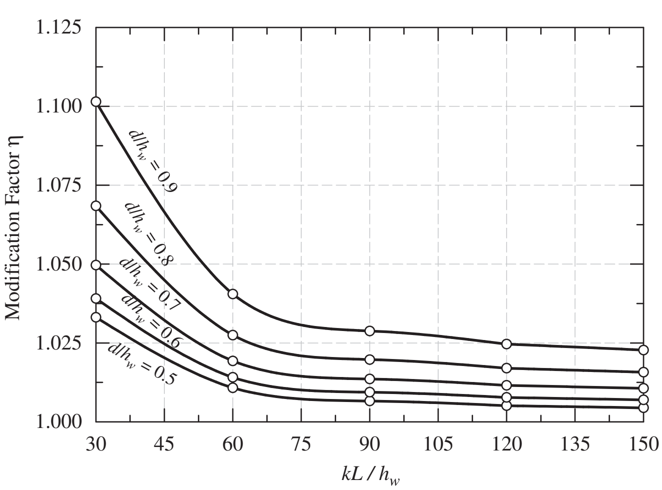

Fig. 9. Variation of the buckling factor η with column length.

Fig. 9 presents sample results that show the variation of the buckling modification factor η with various (kL/h_w) ratios for columns having 450 mm × 10 mm web and 400 mm × 16 mm flange. Five different web castellation sizes of $d/h_w = 0.5$, 0.6, 0.7, 0.8 and 0.9 are considered. Plotted results show the trend of variation of the buckling modification factor η with column length as it approaches unity for long columns with significantly high (kL/h_w) ratio that exceeds 150.

6. Summary and conclusions

The current study presents a simplified approach for the assessment of the effect of shear deformations on axially loaded castellated columns and evaluation of the associated buckling load capacity. The finite element method is employed to perform an extensive numerical stability analysis of a wide spectrum of geometric dimensions and boundary conditions of I-shaped castellated steel columns. Results of the analysis are used to identify a non-dimensional parameter that can be used to calculate the buckling length of castellated columns taking into account the influence of web castellation on altering the flexural and shear deformations from those induced in plan-webbed columns with the same geometrical dimensions. Results of the numerical analysis are also used to investigate the variation of the proposed buckling load modification factor with various parameters defining the geometry of the column's cross-section and the configuration of web castellation along the axis of the column. The main conclusions that may be drawn from the study are summarized as follows:

- Results indicate that columns with bigger castellation size, (d/h_w), are associated with higher values for the buckling modification factor η. As such, these columns experience higher reduction in their buckling capacity due to the encountered reduction in the shear stiffness. For columns with smaller (d/h_w) ratio, the buckling modification factor η approaches unity which is the typical value for plain-webbed long columns, where the shear deformations are minimal.
- Numerical analyses reveal that web aspect ratio (h_w/t_w) has no impact on the buckling modification factor η. Meanwhile, the buckling modification factor η is found to be linearly proportional to the increase in the flange aspect ratio (b_f/t_f).
- Charts are developed to present the variation of the proposed buckling modification factor η with respect to the relative flange-to-web flexural stiffness (I_f/I_w) for a reference (kL/h_w) value of 30. Developed charts show more reduction in the buckling capacity to be associated with the increase in the relative flexural stiffness (I_f/I_w), for the same gross second moment of area I, due to the associated reduction in the shear stiffness of the web and consequently the entire column. The plots show also that the minimum reduction in the buckling capacity corresponds to an optimum (I_f/I_w) value of 2.0. Such value represents the typical average (I_f/I_w) value for commonly used steel sections in practice.
- The influence of the castellated column length on its buckling capacity is also investigated. A general procedure is proposed to allow for evaluation of the buckling modification factor η for any column length other than the reference (kL/h_w) value of 30. The procedure is based on the combined implementation of the developed η-charts that account for the shear effects, along with the equivalent cross-section approach that considers the reduced flexural stiffness of the equivalent cross-section. The suggested equivalent cross-section properties are evaluated for a virtual section having a uniform web perforation with a size equal to 0.806 of the original castellation size.

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References

[1] Timoshenko SP, Gere JM. Theory of elastic stability. 2nd ed. New York: McGraw-Hill Book Company; 1961.
[2] Engesser F. Ueber die Knickfestigkeit gerader Stiibe (On the buckling resistance of perfect columns). Zeitschnf't des Architekten und Ingenieur Vereins ni Hannover 1889;35:455–62.
[3] Engesser F. Die Knickfestigkeit gerader Stiibe (The buckling resistance of 134 perfect columns). Zentralblatt der Bauverwaltung 1891;11:483–6.
[4] Nanni J. Das Eulersche Knickproblem unter Berucksichtigung der Querkrafte (English translation: the effect of shear on the Euler buckling problem). Zeitschrift fur Angewandte Mathematik und Physik 1971;22(1):156–85.
[5] Ziegler H. Arguments for and against Engesser's buckling formulas. Ingenieur-Archiv 1982;52(1/2):105–13.
[6] Gjelsvik A. Stability of built-up columns. ASCE Journal of Engineering Mechanics 1991;117(6):1331–45.
[7] Timoshenko SP. On the correction for shear of the differential equation of transverse vibrations of prismatic bars. Philosophical Magazine (6th Series) 1921;41(245):744–6.
[8] Wang CM. Timoshenko beam-bending solutions in terms of Euler-Bernoulli solutions. ASCE Journal of Engineering Mechanics 1995;121(6): 763–5.
[9] Galambos TV, editor. Guide to stability design criteria of metal structures (4th edition). New York: Wiley Inc.; 1988 Structural Stability Research Council.
[10] McCormac JC. Structural steel design. 4th ed. Upper Saddle River, New Jersey: Pearson Prentice Hall; 2008.
[11] Elmahdy, Ghada M. Buckling of built-up Columns, Ph.D., University of Windsor, Ontario, Canada, 1997.
[12] American Institute of Steel Construction (AISC). Steel Construction Manual. 13th edition., 2005.
[13] British Standard (BS5950-1:2000). Structural use of Steelwork in Building–Part 1: Code of practice for design rolled and welded sections, 2001.
[14] ANSYS User's Manual Release 11.0, Canonsburg, Pennsylvania, 2007.
[15] European Committee for Standardization, CEN, EN 1993-1-1: Design of steel structures—Part 1-1: General rules and rules for building, 2005
[16] European Committee for Standardization, CEN, ENV 1993-1-1:1992/prA2-Annex N: Openings in webs, 1994.
[17] Koiter WT, Kuiken GDC. The interaction between local buckling and overall buckling on the behavior of built-up columns. Report no. 447, Laboratory of Engineering Mechanics, Delft University of Technology, Delft, Holland, 1971.